第11章 结构顺风向随机风振响应
Chapter 11 Alongwind random vibration response of structure

11.1 结构的风振响应的运动方程

11.1 the equation of motion of structure under fluctuating wind

- 作用在结构上的顺风向风压可以分解为：平均风、脉动风。平均风作用下的平均响应$\bar{x}(t)$采用静力计算方法确定。
- Downwind wind pressure acting on the structure can be divided into mean wind and fluctuating wind. The mean response under mean wind action, $\bar{x}(t)$ is calculated by static method.
- 脉动风作用下的脉动响应方差σ_x采用随机振动的方法确定。
- The fluctuating response variance σ_x is determined by random vibration method
- 结构中总的响应 Total structural response：

$$x = \bar{x} + g\sigma_x$$

式中g为峰值因子（peak gust factor）

11.1 结构的风振响应的运动方程

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- The equation of motion of structure under wind loads:

$$[M]\{\ddot{x}(t)\}+[C]\{\dot{x}(t)\}+[K]\{x(t)\}=F(t,\{x(t)\},\{\dot{x}(t)\},\{\ddot{x}(t)\})$$

- 方程右边的荷载包括了作用于结构的脉动风荷载以及由结构自身运动而产生的自激力。The right of the equation includes the fluctuating wind load on the structure and the self-excited force produced by the structural motion.
- 尽管结构体系被假定为线性，由于自激力的影响，方程本身仍然为非线性的，对该方程解析求解的困难就在于方程右端项的不确定性。Although the structural system is considered to be linear, because of the influence of self-excited force, the equation itself is non-linear. The difficulty to solve the equation is the uncertainness of the right of equation

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研究表明：自激力项可以通过气动阻尼的方式来考虑，通常的做法：在结构阻尼比上叠加气动阻尼比，因此结构的运动方程可写为：

Studies shows that: self-excited force may be considered by way of aerodynamic damping. generally the structural damping ratio superimposed on the aerodynamic damping ratio, so the structure of the equations of motion:

$$[M]\{\ddot{x}(t)\}+[C]\{\dot{x}(t)\}+[K]\{x(t)\}=\{p(t)\}$$

在时域内求解时，方程右端为脉动风荷载时程；在频域内求解时，方程右端为脉动风荷载的率谱密度函数。

When solving the equation in the time domain, the right of the equation is the time history of wind load. When solving it in the frequency domain, the right of the equation is the spectral density function of fluctuating wind loads.

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$$[M]\{\ddot{x}(t)\}+[C]\{\dot{x}(t)\}+[K]\{x(t)\}=\{p(t)\}$$

- 时域求解：时程分析、基于振型分解的时程分析

Solution in Time domain : Time-history analysis, time - history analysis based on modal decomposition

P(t) → x(t)

- 频域求解：多自由度随机振动分析、基于振型分解的频域分析

Solution in frequency domain: random vibration analysis of MDOF system, and frequency domain analysis based on modal decomposition

Sp(n) → Sx(n)

11.2 结构的风振响应的时程分析

11.2 time-history analysis of the equation of motion

时程分析 Time history analysis

■也称直接积分法，实质数值计算方法

It is also known as direct integral method, actually an numerical methods

■按假定的加速度变化规律时程分析法可分为 there are different time history method based on the assumption of acceleration variation:

- linear acceleration method
- Wilson-θ method
- Newmark-β method
- Rang-kuta method

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- 多自由度随机振动分析:

$$\left[S_x(\omega)\right]=\left[H(\omega)\right]\left[S_p(\omega)\right]\left[H^*(\omega)\right]$$

- 荷载功率谱矩阵

$$\left[S_p(\omega)\right]=\begin{bmatrix} S_{p_{11}}(\omega) & S_{p_{12}}(\omega) & \cdots & S_{p_{1n}}(\omega) \\ S_{p_{21}}(\omega) & S_{p_{22}}(\omega) & \cdots & S_{p_{2n}}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ S_{p_{n1}}(\omega) & S_{p_{n2}}(\omega) & \cdots & S_{p_{nn}}(\omega) \end{bmatrix}$$

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- 频率响应函数矩阵

$$[H(\omega)] = \begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) & \cdots & H_{1n}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) & \cdots & H_{2n}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ H_{n1}(\omega) & H_{n2}(\omega) & \cdots & H_{nn}(\omega) \end{bmatrix}$$

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- 基于振型分解的频域方法 Frequency domain method based on modal decomposition:

（1）采用振型分解，将多自由运动方程转换为n个单自由度运动方程；

By using modal decomposition，decouple the MDOF equation of motion of MDOF system into n number of equations of motion of SDOF system

（2）采用频域法对单个自由度运动方程进行求解；

Solve SDOF equation of motion using frequency domain method

（3）采用叠加原理得到原结构的解。

superpose the solutions of SDOF system to obtain the solution of original structure.

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模态向量

振型分解

$$[M]\{\ddot{x}(t)\}+[C]\{\dot{x}(t)\}+[K]\{x(t)\}=\{p(t)\}$$

$$\{x(t)\}=\sum_{j=1}^{n}\{\phi_j\}q_j(t)=[\Phi]\{q(t)\}$$

$$[M][\Phi]\{\ddot{q}(t)\}+[C][\Phi]\{\dot{q}(t)\}+[K][\Phi]\{q(t)\}=\{p(t)\}$$

$$[\Phi]^T[M][\Phi]\{\ddot{q}(t)\}+[\Phi]^T[C][\Phi]\{\dot{q}(t)\}+[\Phi]^T[K][\Phi]\{q(t)\}=[\Phi]^T\{p(t)\}$$

$$[\Phi]^T[M][\Phi]=[I]$$

$$[\Phi]^T[K][\Phi]=[\Omega]=diag(\omega_j^2)$$

$$[\Phi]^T[C][\Phi]=diag(2\zeta_j\omega_j)$$

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$$diag(1)\{\ddot{q}(t)\}+diag(2\zeta_j\omega_j)\{\dot{q}(t)\}+diag(\omega_j^2)\{q(t)\}=[\Phi]^T\{p(t)\}$$

$$\begin{bmatrix}1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1\end{bmatrix}\{\ddot{q}(t)\}+\begin{bmatrix}2\zeta_1\omega_1 & & & \\ & 2\zeta_1\omega_1 & & \\ & & \ddots & \\ & & & 2\zeta_n\omega_n\end{bmatrix}\{\dot{q}(t)\}+\begin{bmatrix}\omega_1^2 & & & \\ & \omega_2^2 & & \\ & & \ddots & \\ & & & \omega_n^n\end{bmatrix}\{q(t)\}$$

$$=[\Phi]^T\{p(t)\}$$

$$\ddot{q}_j(t)+2\zeta_j\omega_j\dot{q}_j(t)+\omega_j^2 q_j(t)=\{\phi_j\}^T\{p(t)\}=f_j(t)$$

模态力

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位移响应根方差 Standard deviation of displacement response

$$\ddot{q}_j(t) + 2\zeta_j\omega_j\dot{q}_j(t) + \omega_j^2 q_j(t) = \{\phi_j\}^T \{p(t)\} = f_j(t)$$

$$S_{q_j}(\omega) = H_j(\omega) S_{f_j}(\omega) H_j^*(\omega)$$

$$= H_j(\omega)\{\phi_j\}^T \left[S_p(\omega)\right]\{\phi_j\} H_j^*(\omega)$$

$$H_j(\omega) = \frac{1}{\omega_j^2 - \omega^2 + i \cdot 2\zeta\omega_j\omega}$$

$$f_j(t) = \{\phi_j\}^T \{p(t)\}$$

$$\left[S_q(\omega)\right] = [H(\omega)][\Phi]^T \left[S_p(\omega)\right][\Phi]\left[H^*(\omega)\right]^T$$

模态向量

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位移响应根方差 Standard deviation of displacement response

$$\left[S_q(\omega)\right]=\left[H(\omega)\right][\Phi]^T\left[S_p(\omega)\right][\Phi]\left[H^*(\omega)\right]^T$$

$$\left[S_x(\omega)\right]=[\Phi]\left[S_q(\omega)\right][\Phi]^T$$

$$\{x(t)\}=\sum_{j=1}^{n}\{\phi_j\}q_j(t)=[\Phi]\{q(t)\}$$

$$\left[S_x(\omega)\right]=[\Phi]\left[H(\omega)\right][\Phi]^T\left[S_p(\omega)\right][\Phi]\left[H^*(\omega)\right]^T[\Phi]^T$$

$$\sigma_x^2=\int_{-\infty}^{\infty}S_x(\omega)d\omega$$

$$\left[\sigma_x^2\right]=\int_{-\infty}^{\infty}[\Phi]\left[H(\omega)\right][\Phi]^T\left[S_p(\omega)\right][\Phi]\left[H^*(\omega)\right]^T[\Phi]^T d\omega$$

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$$\left[\sigma_x^2\right]=\int_{-\infty}^{\infty}[\Phi][H(\omega)][\Phi]^T\left[S_p(\omega)\right][\Phi]\left[H^*(\omega)\right]^T[\Phi]^T\,d\omega$$

上式同时考虑了结构不同模态之间的耦合作用，称为**CQC**组合

$$\left[\sigma_x^2\right]=\sum_j\left[\sigma_{x_j}^2\right]=\sum_j\int_{-\infty}^{\infty}\{\phi_j\}H_j(\omega)\{\phi_j\}^T\left[S_p(\omega)\right]\{\phi_j\}H_j^*(\omega)\{\phi_j\}^T d\omega$$

如果不同振型之间的耦合作用较弱，不同振型之间的交叉项就可以略去，这时可以只保留角标相同的项，称为**SRSS**组合

$$\left[\sigma_x^2\right]=\int_{-\infty}^{\infty}[\Phi][H(\omega)][\Phi]^T\left\lfloor S_p(\omega)\right\rfloor[\Phi]\left[H^*(\omega)\right]^T[\Phi]^T\,d\omega$$

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- 模态位移法所求近似解得精度取决于参振模态$\{\phi_i\}$（i=1, ...,m）的选取，若参加叠加的模态阶数不够时，近似解将会产生较大的误差

The accuracy of the approximate solution by modal displacement method depends on the selection of vibration modes $\{\phi_i\}$（i=1, ...,m）. There will be large error when the superstitious modes are insufficient.

- 误差的大小取决于体系能否激起被舍去的高阶模态，而这些高阶模态能否产生较大的振动

高层、高耸建筑物的风振响应可以近似只考虑第一阶阵型。

The size of error depends on whether the system can excite the excluded higher modes and whether the higher modes can produce large vibration.

$$\left[\sigma_x^2\right] \approx \int_{-\infty}^{\infty} \{\phi_1\} H_1(\omega) \{\phi_1\}^T S_p(\omega) \{\phi_1\} H_1^*(\omega) \{\phi_1\}^T d\omega$$

- 高层、高耸建筑物的风振响应可以近似只考虑第一阶阵型。

For high-rise buildings, it is enough to just consider the first mode.