第四章 结构被动调谐减震控制
Chapter 4 Structural passive tuned vibration control

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4. 1　结构被动调谐减震控制概述

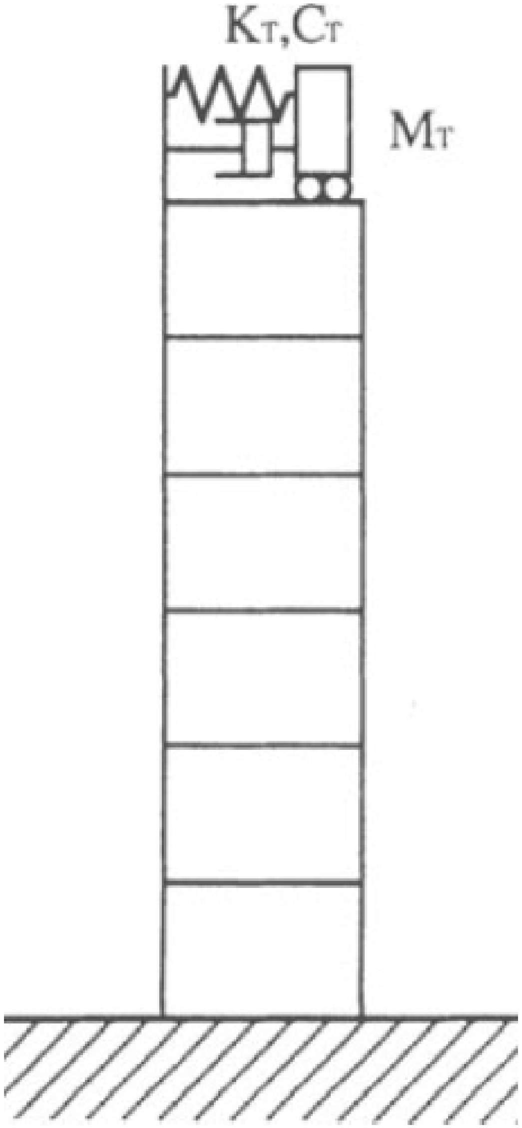
Introduction to passive tuned vibration control

4. 1. 1 Principles of passive tuned vibration control

- Principles

　　结构被动调谐减震控制体系，由结构和附加在主结构上的子结构组成。附加的子结构具有质量、刚度和阻尼，因而可以调整子结构的自振频率，使其尽量接近主结构的基本频率或激振频率。这样，当主结构受激励而振动时，子结构就会产生一个与结构振动方向相反的惯性力作用在主结构上，使主结构的振动反应衰减并受到控制。由于这种减震控制不是通过提供外部能量，只是通过调整结构的频率特性来实现的，故称为“被动调谐减震控制”。也称“动力消震”。

Passive tuned vibration control system consists of main structure and the substructure attached to it. The additional substructure has mass, stiffness and damping, so it is possible to tune the natural period of substructure by itself with the fundamental frequency of the main structure. Thus, when the main structure vibrates under excitation, the substructure will produce an inertia force acting on the main structure whose direction is opposite to the vibration of the main structure, which minimize the vibration response of the main structure. Since this kind of vibration control is achieved by tuning the structural frequency and does not need any external energy supply for operation, it is called passive tuned vibration control , also known as “dynamic vibration absorption”.



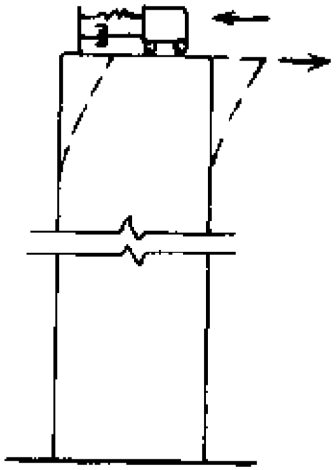
- Types of passive tuned damper

调谐质量阻尼器(Tuned Mass Damper简TMD)：

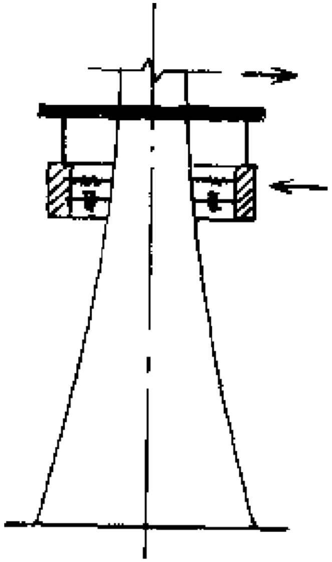
支承式调谐质量阻尼器　Translational TMD

悬吊式调谐质量阻尼器　Suspending TMD

悬吊式调谐摆动阻尼器Pendulum Tuned Mass Damper



(a)



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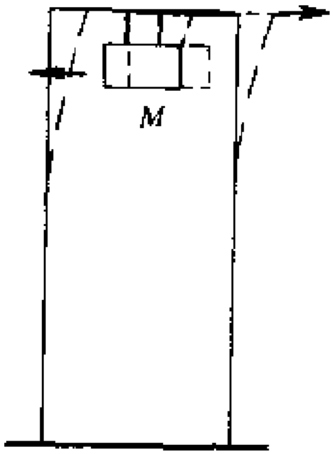


(c)

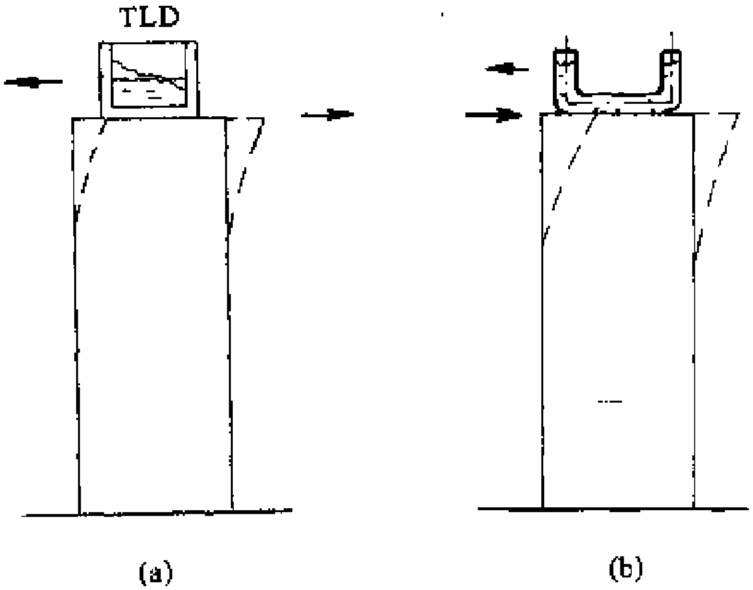
- Types of passive tuned control
  - 调谐液体阻尼器（Tuned Liquid Damper, 简TLD）：

    调谐液体储液池式阻尼器 sloshing damper

    谐液体U形柱式阻尼器 tuned liquid column damper

First is the sloshing damper as shown in Fig. (a). Vibration period is adjusted by the size of the container or depth of the liquid. Damping capacity is increased by placing meshes or rods in the liquid.

The second is the tuned liquid column damper as shown in Fig, (b). Vibration period is adjusted by shapes of column or air pressure in the column. Damping capacity is increased by adjusting the orifice in the column which generates high turbulence.



4.1.２结构被动调谐减震控制的优越性和应用范围 Advantages and applications of structural passive tuned vibration control

- Advantages

（1）有效衰减结构的振动反应；Effectively reduce the vibration response of structure

（3）使用范围广；节省结构造价 the wide range of application, cost-saving of the structure

（3）为某些重大结构的减震提供难以代替的途径； providing some important structures with irreplaceable ways of vibration absorption

（4）不仅适用于新建结构的减震控制，也特别适用于已有结构的减震改良；Not only suitable to the vibration control of the new structures but also the improvement of the existed structures

（5）可充分利用已有结构的部件或设备作为子结构，不必专设；Making the full use of the existing structure members or devices as substructure

（6）一次性装设，永久使用，无须维修调换，比较符合工程应用的实际情况。

Once installed, they can be used permanently without repair or replacement

4.1.２结构被动调谐减震控制的优越性和应用范围 Advantages and

- 应用范围：applications

（１）多层、高层、超高层建筑；

multi-storey, high-rise, and super high-rise buildings;

（２）高耸塔架、烟囱结构等；

high-rise towers, chimneys and other structures;

（３）大跨度桥梁；

large-span bridges;

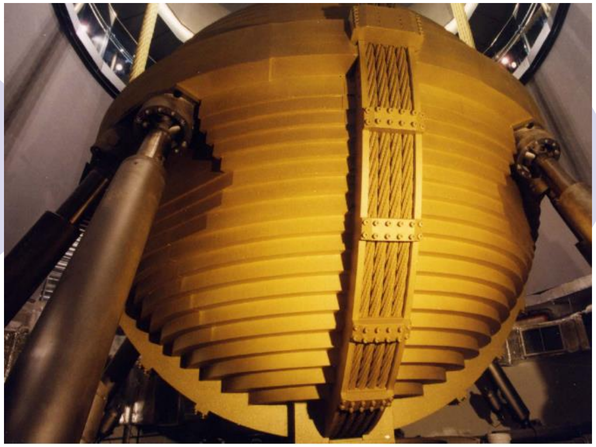
（４）海洋平台或其他特种结构；

offshore platform or other special structures;

（５）已有建筑物的“加层减震”。

Adding vibration absorption layers on the existed buildings

大楼受风吹袭时会微幅摆动，会使大楼里的人有晕眩之感，阻尼器可以减小微幅摆动，提高人的舒适感。该阻尼器由41层12. 5cm厚的实心钢板堆叠焊接而成，重达730吨。



The building vibrate slightly under wind excitation for long duration, which makes people inside dizzy. To increase comfort of working people and function of installed machineries and equipments, the dampers are needed. The TMD was made of 41 layers of 12.5cm steel plates welded together, over 730 ton weight.

Rendering of Taipei 101 Building TMD

台北101大楼风阻尼器
Wind dampers of Taipei 101 Building

上海环球金融中心风阻尼器
Wind Damper in Shanghai World Financial Center

4. 2 单质点结构被动调谐减震控制

Vibration control of SDOF system with TMD(TLD)

对于多质点体系的剪切形结构在地震或风作用下，其第一振型的振动反应占结构总反应约80%以上。因而，对于某些多质点结构体系而言，在工程上有时可以近似简化为具有基本振型的等效单质点体系进行分析。

For a shear MDOF structural system under earthquake or wind action, the vibration response of the first mode account for more than about 80% of the total response of structure. Thus in engineering practice, some MDOF structural system can be approximately simplified as an SDOF system with the same fundamental mode.

4.2 单质点结构被动调谐减震控制

Vibration control of SDOF system with TMD

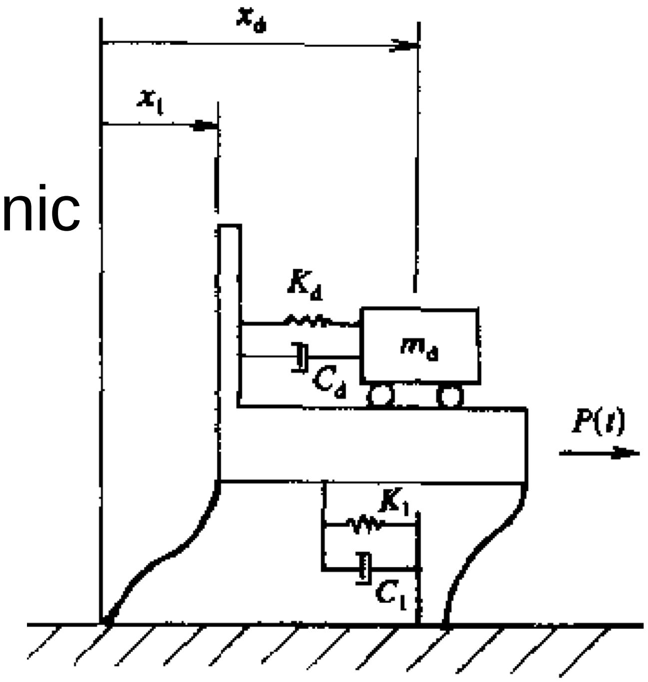
主结构受外激励的情况分两种，一种是直接对主结构激励（例如风、海浪、机器振动等），另一种是通过地面激励（例如地震、环境振动等）。外激励又分为两类，一类是简谐激励（机器振动），另一类是随机激励（地震、风、海浪等）。

The are two types of excitation on the structure. one is the excitation directly on the main structure (such as wind, waves, machine vibration, etc.), and the other is ground excitation (such as earthquakes, environmental vibration, etc.). According to the frequency spectra, the external excitations can be classified as the harmonic excitation (vibration machine), and the random excitation (earthquake, wind, waves, etc.)

4. 2. 1 主结构直接受简谐激励的调谐减震控制 Tuned vibration absorption system under direct harmonic excitation

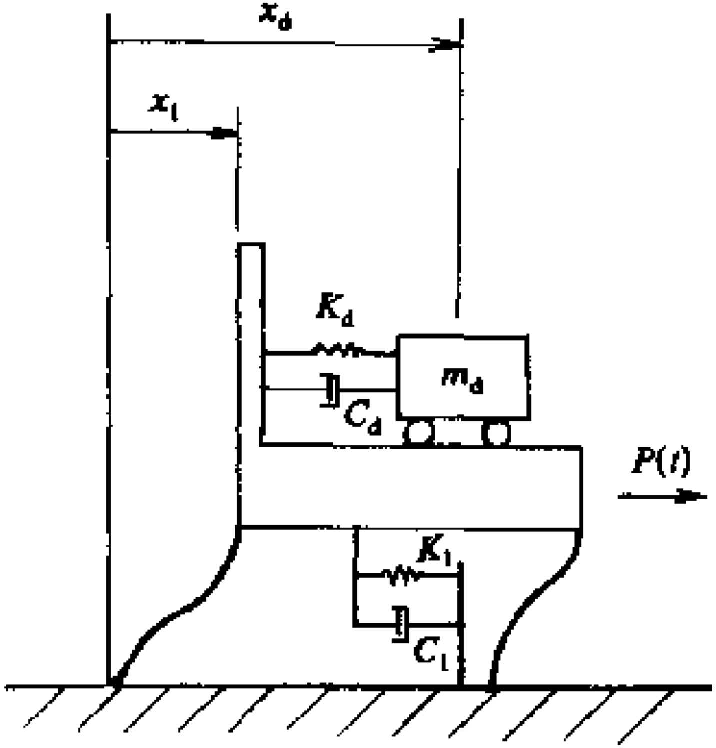
Passive tuned vibration absorption is adopted when the machines in the structure vibrate and directly act harmonic excitation on the main structure.

The dynamic model of SDOF system passive tuned vibration absorber is showed in the pictures



4. 2. 1 主结构直接受简谐激励的调谐减震控制 Tuned vibration absorption system under direct harmonic excitation

- m_1, K_1, C_1 are the mass, stiffness, and damping of main structure respectively.
- m_d, K_d, C_d are the mass, stiffness, and damping of the substructure respectively.
- P(t) is the external harmonic excitation, $P(t)=P_0\sin\omega t$
- x_1 is the displacement of main structure
- x_d is the displacement of substructure



无阻尼子结构的调谐减震控制 Vibration control of undamped structure with undamped TMD

$$m_1\ddot{x}_1 + (K_1 + K_d)x_1 - K_d x_d = P(t)$$

$$m_d\ddot{x}_d + K_d(x_d - x_1) = 0$$

Calculating by the conversion function method(转换函数法)

$$x_1 = H_1(\omega)P_0 = \frac{P_0}{K_1}\,\frac{f^2 - h^2}{h^4 - h^2[1 + f^2(1 + \mu)] + f^2}$$

$$x_d = H_d(\omega)P_0 = \frac{P_0}{K_1}\,\frac{f^2}{h^4 - h^2[1 + f^2(1 + \mu)] + f^2}$$

$$A_1 = \frac{f^2 - h^2}{h^4 - h^2[1 + f^2(1+\mu)] + f^2},$$

$$A_d = \frac{f^2}{h^4 - h^2[1 + f^2(1+\mu)] + f^2}$$

$$A_1 = \frac{f^2 - h^2}{h^4 - h^2[1 + f^2(1+\mu)] + f^2}, \quad A_d = \frac{f^2}{h^4 - h^2[1 + f^2(1+\mu)] + f^2}$$

$$f = \frac{\omega_d}{\omega_1}, \qquad h = \frac{\omega}{\omega_1}, \qquad \mu = \frac{m_d}{m_1}$$

式中，h:外激励与主结构的频率比，μ:子结构与主结构的质量比，f: 子结构与主结构的固有频率比

Where，h is the ratio of the frequency of external excitation to the natural frequency of main structure.

μ is the mass ratio of substructure to main structure.

f is the natural frequency ratio of substructure to main structure.

（1） when the natural frequency of substructure ω_d equals the that of the excitation ω, that is $\omega_d = \omega$, then $f = h$.

$$x_1 = \frac{P_0}{K_1}A_1 = 0$$

$$x_d = \frac{P_0}{K_1}A_d = -\frac{P_0}{K_d}$$

$$(x_d - x_1)K_d = -P_0$$

x_1=0表明，当主结构直接被简谐激励振动时，使主结构达到最优调谐减震的条件：子结构的固有频率等于激励频率$\omega_{d=}\omega$；

当满足上述条件时，子结构向主结构施加一个惯性力，其大小与激励外力相等，方向相反，使主结构的振动反应消失，这就是被动调谐减震的机理

x_1=0 shows that when the main structure is under direct harmonic excitation, in order to achieve the optimal tuned vibration absorption on main structure , the natural frequencyω_d of substructure must be equal that of the excitation ω. When $\omega_{d=}\omega$, an inertia force is exerted on the main structure by the substructure to make the structural vibration disappear, which equals to the value of external excitation force with an opposite direction. And this is the mechanism of passive tuned vibration absorption

$$A_1 = \frac{f^2 - h^2}{h^4 - h^2[1+f^2(1+\mu)] + f^2}, \quad A_d = \frac{f^2}{h^4 - h^2[1+f^2(1+\mu)] + f^2}$$

(2) When the frequency of substructure is equal to the natural frequency of main structure, $\omega_d = \omega_1$, that is, f=1,

if the main structure resonate with external excitation, that is:

$$\omega = \omega_1,\ h=1$$

The displacement x_1 is zero:

$$x_1 = \frac{P_0}{K_1} A_1 = 0$$

$x_1=0$表明:当主结构被外激励触发共振时，使主结构达到最优调谐减震的条件：子结构的固有频率等于主结构的固有频率，$\omega_d = \omega_1$；当$\omega_d = \omega_1$时，f=1，对不同的μ值，可以得到不同的A_1-h曲线。

$x_1=0$ shows that when the resonance of the main structure was triggered by the external excitation, in order to achieve the optimal tuned vibration absorption on main structure, the natural frequency of substructure ω_d should equals to that of the main structure ω_1 , $\omega_d = \omega_1$ and f=1. The different A_1-h curves can be obtained with different μ.

$$A_1 = \frac{f^2 - h^2}{h^4 - h^2[1+f^2(1+\mu)] + f^2}, \quad A_d = \frac{f^2}{h^4 - h^2[1+f^2(1+\mu)] + f^2}$$

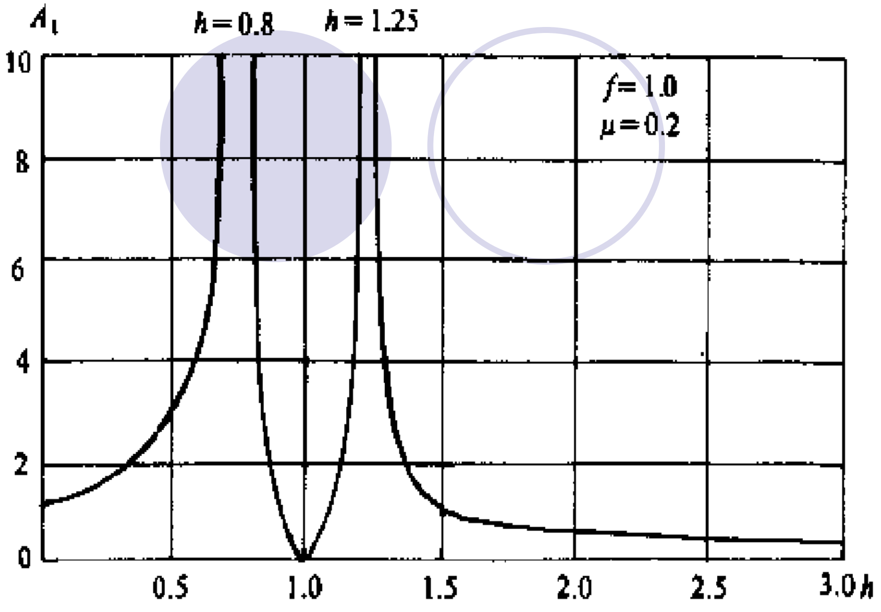


图 14-1-2 无阻尼子结构体系的 A_1-h 的关系曲线

h=1时，$A_1=0$, $\omega_d=\omega_1=\omega$，结构达到最优控制效果。但当h在1附近时，左右两侧有两个峰值，说明若激励频率偏离子结构的固有频率，主结构可能会产生很大的振动反应，因此对无阻尼子结构的调谐减震体系，只能适用于外激励频率稳定的情况，为了使减震体系适应宽频带外激励，可装多个子结构。

When $h=1$, $A_1=0$, that is: $\omega_d=\omega_1=\omega$, the structure achieves the optimal control effects. But there are two peaks on the both sides of the h=1, which indicates that if the excitation frequency deviates from the structural natural frequency, the main structure may produce large vibration response. Therefore the tuned vibration absorption system with an undamped substructure is only applicable for the stable external excitation frequency. In order to make the vibration absorption system applicable to a broad band of frequency of the external excitation, multiple substructures can be installed.

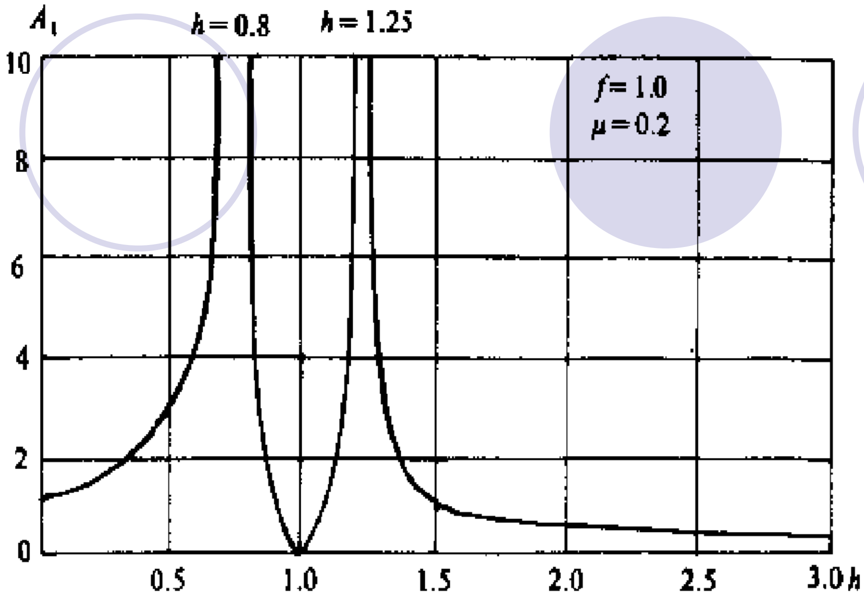


图 14-1-2　无阻尼子结构体系的 A_1-h 的关系曲线

当$h>2.5$时，A_1趋于0，说明调谐减震体系特别适用于固有频率较低的高柔结构，如高层建筑，高耸塔架等，而不适用于固有频率较高的刚性结构

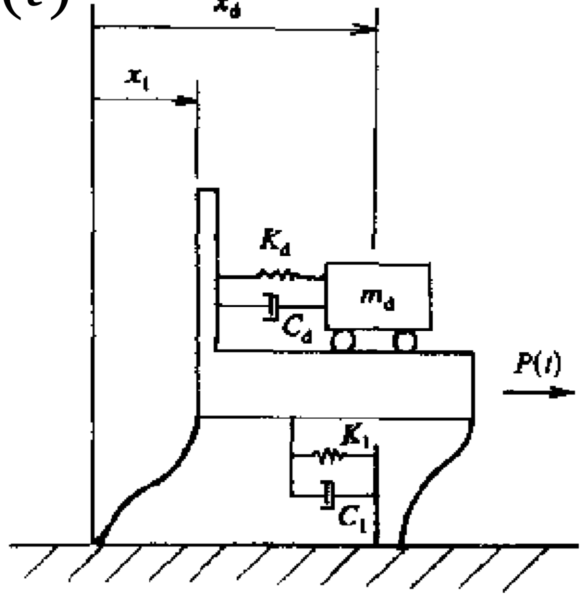
When $h>2.5$, A_1 tends to zero, which indicates that tuned damping system is especially suitable for high and flexible structures with low natural frequencies, such as high-rise buildings, high-rise tower, etc. but not rigid structure with high natural frequencies.

有阻尼子结构的调谐减震控制 Vibration control of structure with TMD

The equation of motion of the SDOF system neglecting the damping of main structure, that is $C_1=0$:

$$m_1\ddot{x}_1 + C_d(\dot{x}_1-\dot{x}_d)+(K_1+K_d)x_1-K_d x_d = P(t)$$

$$m_d\ddot{x}_d + C_d(\dot{x}_d-\dot{x}_1)+K_d(x_d-x_1)=0$$



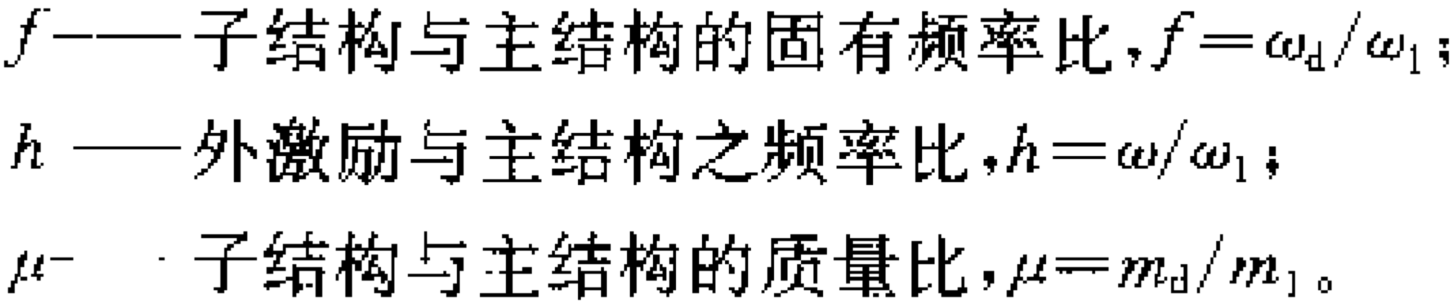
同理用转换函数的方法进行求解可得主体结构位移反应及动力放大系数：

Similarly,, the displacement of main structure and dynamic amplification factor can be solved by the transfer function method:

$$x_1=\frac{P_0}{K_1}A_1$$

$$A_1=[\frac{(2\xi_d h)^2+(h^2-f^2)}{(2\xi_d h)^2(h^2-1+\mu h^2)+[\mu f^2 h^2-(h-1)(h^2-f^2)]^2}]^{\frac{1}{2}}$$

$$A_1 = \left[\frac{(2\xi_d h)^2 + (h^2 - f^2)}{(2\xi_d h)^2 (h^2 - 1 + \mu h^2) + [\mu f^2 h^2 - (h-1)(h^2 - f^2)]^2}\right]^{\frac{1}{2}}$$



f——子结构与主结构的固有频率比，$f=\omega_d/\omega_1$；

h——外激励与主结构之频率比，$h=\omega/\omega_1$；

μ——子结构与主结构的质量比，$\mu=m_d/m_1$。

Where, f is the natural frequency ratio of the substructure to main structure, $f=\omega_d/\omega_1$

h *is* the ratio of frequency of external excitation to the natural frequency of main structure $h=\omega/\omega_1$

μ *is* the mass ratio of the substructure to the main structure, $\mu=m_d/m_1$

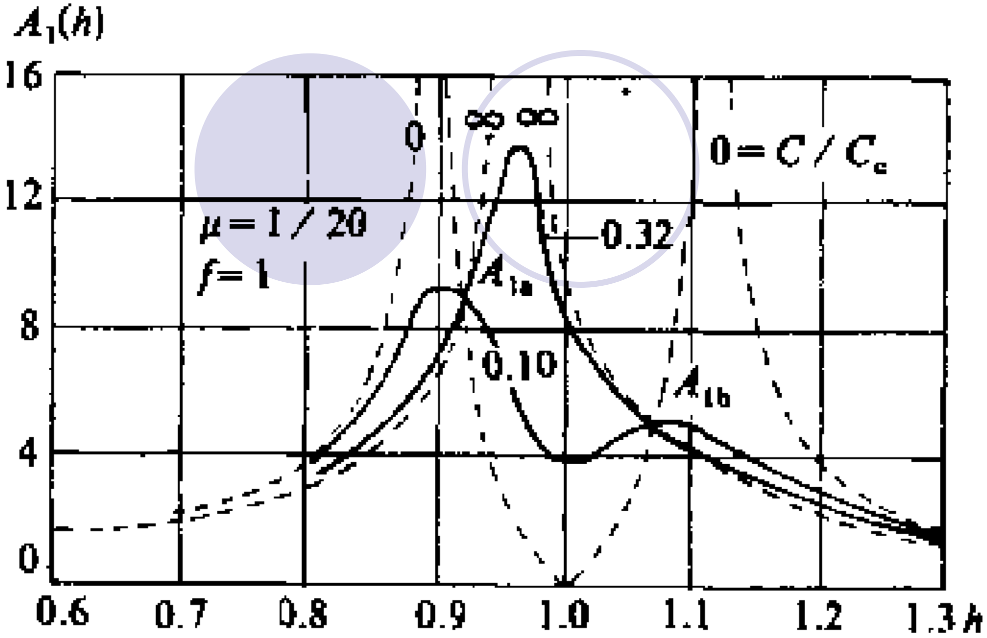


图 14-1-3　有阻尼子结构的 A_1 与 h 的关系曲线

（图中数字代表 ξ_d 值）

whenξ_d=0, it is the A_1-h curve for tuned system with undamped TMD

Whenξ_d=∞, it is the the A_1-h curve for tuned system for conventional structure.

whenξ_d=0.32, it is the curve for energy dissipating structural system with large damping. because the relative displacement occurs between the substructure and main structure, the energy dissipation is more prominent than the tuning effect

whenξ_d=0.10, it is the curve for the tuned vibration absorption system with damped TMD,

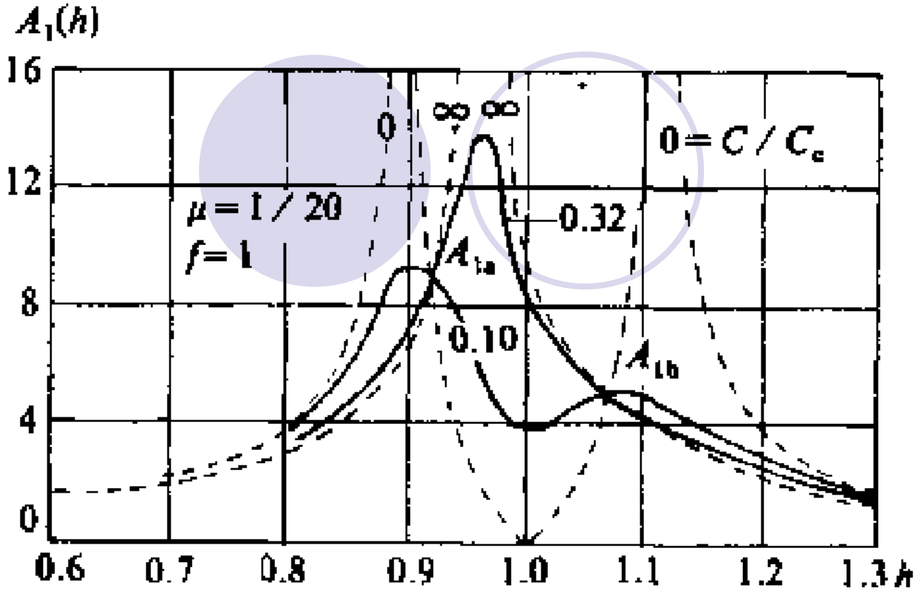


图 14-1-3　有阻尼子结构的 A_1 与 h 的关系曲线

（图中数字代表 ξ_d 值）

For the ξ_d=0.10 curve, there are three features: (1) there are two peak on the both sides of h=1; (2) the peak values are much less than that of conventional structure; (3) the curve groups intersect at two same points.

- 为了使调谐减震结构具有最优减震效果，可以通过选择最优频率比及最优阻尼比，使主结构的位移反应最小。
- The optimal tuned vibration absorption structure can be achieved by selecting the optimal frequency ratio and optimal damping ratio to make the displacement response of main structure minimum.

由图及A_1的计算公式可知，不同的f、μ、ξ_b对应不同的A_1-h曲线，如果要得到A1的最小峰值：尽量使 1）曲线的两个峰值相等；2）曲线的一个峰值与曲线群的公共交叉点重合。

.The figure and formula of A_1shows that: the different f, μ, and ξ_b correspond to the different A_1-h curves. To get the minimum peak of A_1, the following conditions should be met:

1) The two peaks value should be equal;
2) one peak value coincides with one of the intersection points of the curve groups.

when the above conditions are met, the optimal frequency ratio and optimal damping ratio can be obtained：

$$f_{opt} = 1/(1+\mu) \qquad \xi_{opt} = \sqrt{3\mu/8(1+\mu)}$$

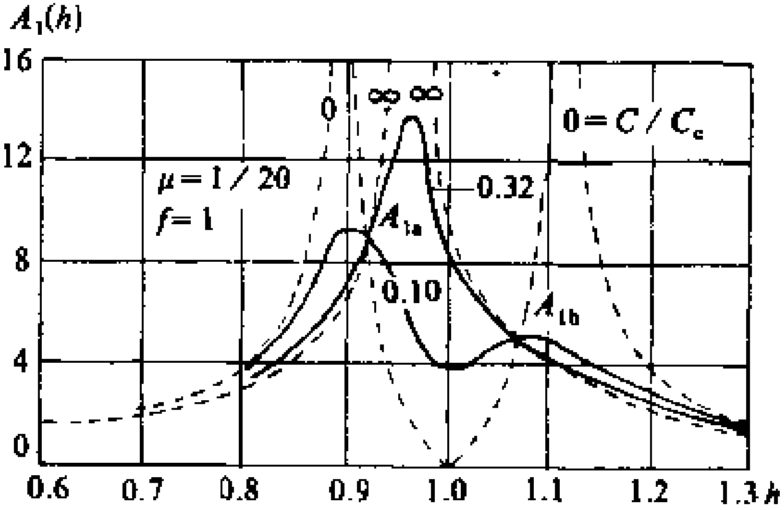


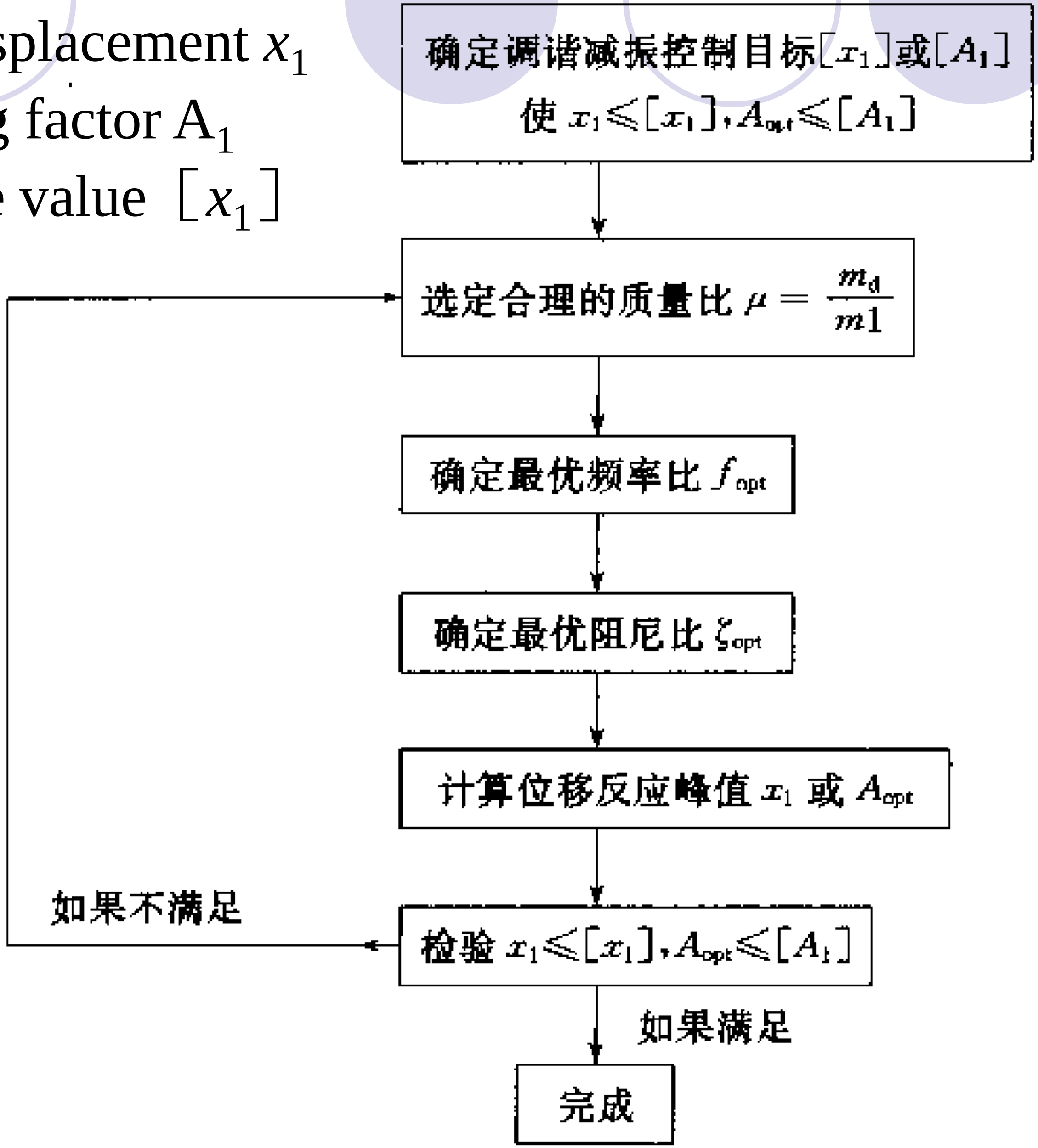
图 14-1-3 有阻尼子结构的 A_1 与 h 的关系曲线
（图中数字代表 ξ_d 值）

Correspondently minimum displacement amplifying factor can be solved:

$$A_{opt} = \sqrt{\frac{2+\mu}{\mu}}$$

Design of TMD system under direct harmonic excitation

Target: to make the displacement x_1 or dynamic amplifying factor A_1 less than the allowable value $[x_1]$ or $[A_1]$

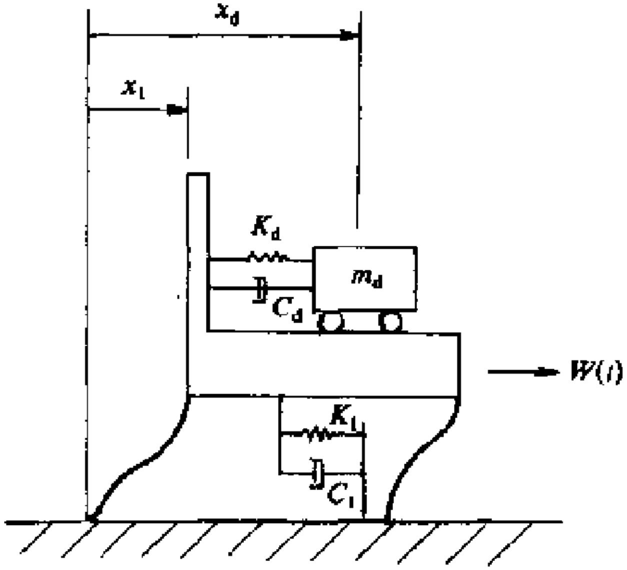


4. 2. 2 随机激励下的调谐减震控制

Tuned vibration absorption system under random excitation

风、海浪等对结构的作用，可按主结构直接受随机激励考虑，采用被动调谐减少主结构的振动反应。

The action by wind, sea waves on the structure can be treated as the direct excitation, the dynamic response of main structure can by reduced by adopting passive TMD or TLD.



Equations of motion :

$$m_1\ddot{x}_1+(C_1+C_d)\dot{x}_1+(K_1+K_d)x_1-C_d\dot{x}_d-K_dx_d=W(t)$$

$$m_d\ddot{x}_d+C_d(\dot{x}_d-\dot{x}_1)+K_d(x_d-x_1)=0$$

4. 2. 2 随机激励下的调谐减震控制

Tuned vibration absorption system under random excitation

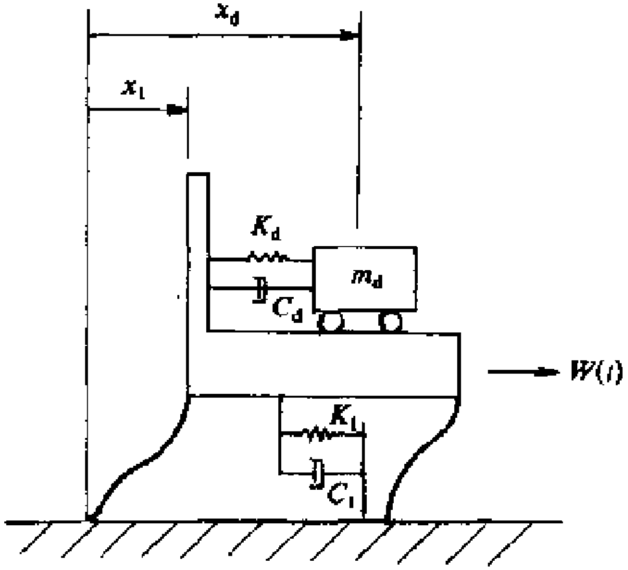
Dynamic model is shown in the picture.

Where,

m_1, K_1, C_1 are the mass, stiffness, and damping of the main structure respectively.

m_d， K_d， C_d are the mass, stiffness, and damping of the substructure respectively.

W(t) is the random external excitation.



为求主结构的位移反应x_1，可采用传递函数法。主结构的位移反应$x_1(t)$与激振力W(t)的传递函数为G(ω),即

The transfer function method can be used to solve the displacement response x_1 of main structure. Assuming G (ω) is the transfer function of displacement response $x_1(t)$ of main structure and the excitation W(t), that is:

$$G(\omega)= x_1(t)/ W(t)$$

设随机激振为白噪声随机过程，其频谱密度为S_0，则主结构位移反应x_1的频谱密度为：

Assuming the excitation is the white-noise random process and its spectral density is S_0. Then the spectral density of displacement x_1 is:

$$S_{x_1}(\omega)= G^2(\omega) \cdot S_0$$

主结构位移反应x_1的方差为：

The variance of the displacement response of main structure is:

$$\sigma_{x_1}^2 = \frac{\sigma_0}{2\pi}\int_{-\infty}^{\infty} \left|G(\omega)\right|^2 d\omega$$

$$G(\omega) = (f^2\omega_1^2 - \omega^2 + i2\zeta_d f\omega_1)/Z(\omega)m_1$$

$$Z(\omega) = \omega^4 - [1 + \mu(1 + f^2) + 4\zeta_1\zeta_d]\omega_1^2\omega^2 + f^2\omega_1^4 - i2\omega_1[\zeta_1 + (1 + \mu)\zeta_d f]\omega^3 + i(\zeta_1 + \mu f\zeta_d)2f^2\omega_1^3\omega$$

将将传递函数G (ω)带入位移方差公式中，对位移方差求导，得到其最小值，即可得到最优频率比和最优阻尼比：

To get the minimum value of displacement variance, Substituting the transfer function G (ω) into the formula of displacement variance, and differentiate the formula, the optimal frequency ratio and damping ration are obtained:

$$f_{opt} = (1+0.5\mu)^{1/2} \Big/ (1+\mu) \qquad \xi_{opt} = \left[\frac{\mu(1+0.75\mu)}{4(1+\mu)(1+0.5\mu)}\right]^{-\frac{1}{2}}$$

把以上两值带入位移反应的方差可得最小方差：
The minimum variance of displacement can be obtained by substituting the ζ_{opt} and f_{opt}

$$\sigma^2_{x_1 \min} = \frac{2\pi S_0 \omega_1}{K_1^2}\left[\frac{1+0.75\mu}{\mu(1+\mu)}\right]^{\frac{1}{2}}$$

由上可知：增大子结构的质量比μ，能减少主结构的位移反应x_1的最小方差值，即可提高调谐减震效果。随着质量比μ值的增大，子结构的最优阻尼值 ζ_{opt}也将相应增大。
The minimum variance of the structural displacement response x_1 can be reduced by increasing the mass ratioμ, that is, the tuned vibration absorption effect is improved. With the increment of the mass ratio μ, the optimal damping ratio of the substructure ζ_{opt} will be increased correspondingly.

4.2.3 基底受地震激励的结构的调谐减震控制

Tuned vibration absorption system under earthquake excitation

For the SDOF system with TMD under earthquake excitation, the equations of motion:

$$m_1\ddot{x}_1+(C_1+C_d)\dot{x}_1+(K_1+K_d)x_1-C_d\dot{x}_d-K_dx_d=-m_1\ddot{x}_g$$

$$m_d\ddot{x}_d+C_d(\dot{x}_d-\dot{x}_1)+K_d(x_d-x_1)=-m_d\ddot{x}_g$$

为求得x_1，x_d， 采用传递函数法。设主结构和子结构的位移反应$x_1(t)$及$x_d(t)$与地面地震加速度的传递函数为$H_1(\omega)$及$H_d(\omega)$

The transfer function method can be used to solve the displacement response x_1 and x_d. Assuming $H_1(\omega)$ and $H_d(\omega)$ are the transfer functions of displacement response $x_1(t)$ and x_d and the earthquake acceleration respectively:

To

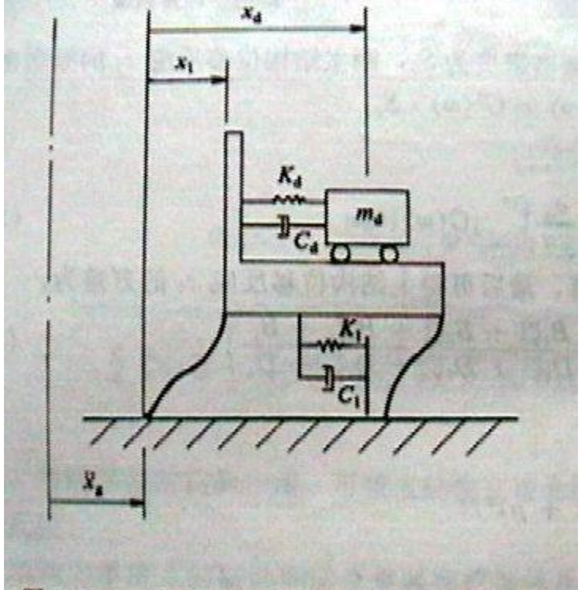


图 14-3-1 单质点体系受地震激励的计算模型

the transfer functions $H_1(\omega)$and $H_d(\omega)$are as follows:

$$H_1(\omega)=\frac{x_1(t)}{\ddot{x}_g(t)};\qquad H_d(\omega)=\frac{x_d(t)}{\ddot{x}_g(t)}$$

为了便于分析，设基底受白噪声随机激励，其频谱密度为S_0，则主结构和子结构的位移反应x_1和x_d的频谱密度为：

To simplify the analysis, assuming that the ground motion is a white-noise random process with an spectral density of $S_{0.}$The spectral densities of the displacement of the main structure, x_1 and that of the substructure, x_d are as follows respectively:

$$S_{x_1}(\omega)=H_1^2(\omega)S_0$$

$$S_{x_d}(\omega)=H_d^2(\omega)S_0$$

主结构和子结构的位移反应x_1和x_d的方差为：

The variances of the displacement response x_1, x_d are respectively

$$\sigma_{x_1}^2 = \frac{S_0}{2\pi}\int_{-\infty}^{\infty} |H(\omega)|^2 d\omega \qquad \sigma_{x_d}^2 = \frac{S_0}{2\pi}\int_{-\infty}^{\infty} |H(\omega)|^2 d\omega$$

经积分运算，可求得主结构位移反应x_1及子结构与主结构相对位$\delta_d = x_1 - x_d$的方差。

The variances of the displacement response x_1 and relative displacement of substructure and main structure $\delta_d = x_1 - x_d$ can be solved by integrating the above formula：

$$\sigma_{x_1}^2 = \frac{\pi S_0 \omega_d^2}{2\zeta_d \mu f^2}\{1 + [4(1+\mu)\zeta_d^2 - (2+\mu)]f^2 + (1+\mu)^2 f^4\}$$

$$\sigma_{\delta_d}^2 \doteq \frac{\pi S_0}{2\mu\omega_d^3\zeta_d}[\mu + (1+\mu)^2 f^2]$$

$$\sigma_{x_1}^2 = \frac{\pi S_0 \omega_d^2}{2\zeta_d \mu f^2}\{1 + [4(1+\mu)\zeta_d^2 - (2+\mu)]f^2 + (1+\mu)^2 f^4\}$$

$$\sigma_{\delta_d}^2 = \frac{\pi S_0}{2\mu \omega_d^3 \zeta_d}[\mu + (1+\mu)^2 f^2]$$

由上式可知：1) 为了减少子结构和主结构相对位移$\delta_d=x_d-x_1$的方差，可以通过提高子结构阻尼比ζ_d予以解决。

The above formula shows that: 1) the damping ratio of the substructure can be increased to reduce the variance of the relative displacement $\delta_d=x_d-x_1$ of the main structure and substructure.

2) 为了使主结构的位移x_1的方差最小，对ζ_d求导数等于零，可得调谐减震体系的子结构最优阻尼比ζ_{opt}。

In order to get the minimum variances of the displacement response of main structure x_1, let the derivative of ζ_d be zero, and the optimal damping ratio ζ_{opt} of the tuned vibration absorbing substructure can be solved.

$$\zeta_{\mathrm{opt}} = \{[1-(2+\mu)f^2+(1+\mu)^2 f^4]/4f^2(1+\mu)\}^{\frac{1}{2}}$$

$$\sigma^2_{x_1\mathrm{min}} = \frac{2\pi S_0\omega_d}{\mu}\{(1-\mu)[1-(2+\mu)f^2+(1+\mu)^2 f^4]\}^{\frac{1}{2}}$$

在进行减震设计时，取$\omega_d = \omega_1$，即f=1， 可以得到相应的主结构位移反应的最小方差和相对位移的最小方差：

When design the tuned vibration absorption structures, Let $\omega_d = \omega_1$, namely, f=1, then the minimum variance of displacement of the corresponding main structure and the relative displacement of substructure can be obtained:

$$f = 1 \qquad \xi_{opt} = 0.5\sqrt{\mu}$$

$$\sigma^2_{x_1\min} = 2\pi S_0\omega_1(1+\mu)/\mu^{\frac{1}{2}}$$

$$\sigma^2_{\delta_1\min} = \pi S_0[(1+\mu)^2+\mu]/2\xi_{opt}\mu\omega_1^2$$

4. 3　多质点结构被动调谐减震控制

Passive tuned vibration control of MDOF system

对于高层建筑、高耸结构、大跨结构等柔性结构，为减小其地震响应，可采用被动调谐减震控制体系。

For high-rise buildings, large-span structures and other flexible structures, passive tuned vibration control system can be adopted to reduce their seismic response.

很多高层建筑或高耸结构等剪切形结构，可以简化为具有多个质点的多自由度体系。采用被动调谐减震控制，即为多质点结构在地震激励下的调谐减震控制。

Many shear structures such as high-rise buildings can be simplified as an multi-degree-of-freedom system, which can adopt passive tuned vibration control to reduce its seismic response

4. 3　多质点结构被动调谐减震控制

Passive tuned vibration control of MDOF system

如果只有一个子结构，则该子结构一般装设于顶层，以控制第一振型的最大位移，从而达到最优减震控制效果。本章讨论在结构顶层装设一个子结构的情况。如果有多个子结构，或装设于其他质点位置，其设计计算方法基本相同。

If there are only one substructure, to achieve the optimal damping control effect, which is generally installed on the top story to control the maximum displacement of the first mode vibration. Our discussion is limited to the case that only one substructure is installed on the top floor of structures. The calculation process is almost the same for the MDOF system with multiple substructures and that with substructures installed at other floor.

A shear high-rise building installed a TMD on the top floor is taken as an example to demonstrate the passive tuned vibration control of MDOF system:

Where, $m_1,m_2,\ldots\ldots,m_i,\ldots,m_n$ are the masses of the floor 1, 2, … I, … n respectively.

$x_1,x_2,\ldots\ldots,x_i,\ldots,x_n$ are respectively the relative displacements between the floor 1, 2, … I, … n and ground

m_d ,K_d and C_d are the mass, stiffness and damping of the substructure respectively

x_d is the relative displacement between substructure and top floor

$\ddot{x}_g$ is the ground acceleration

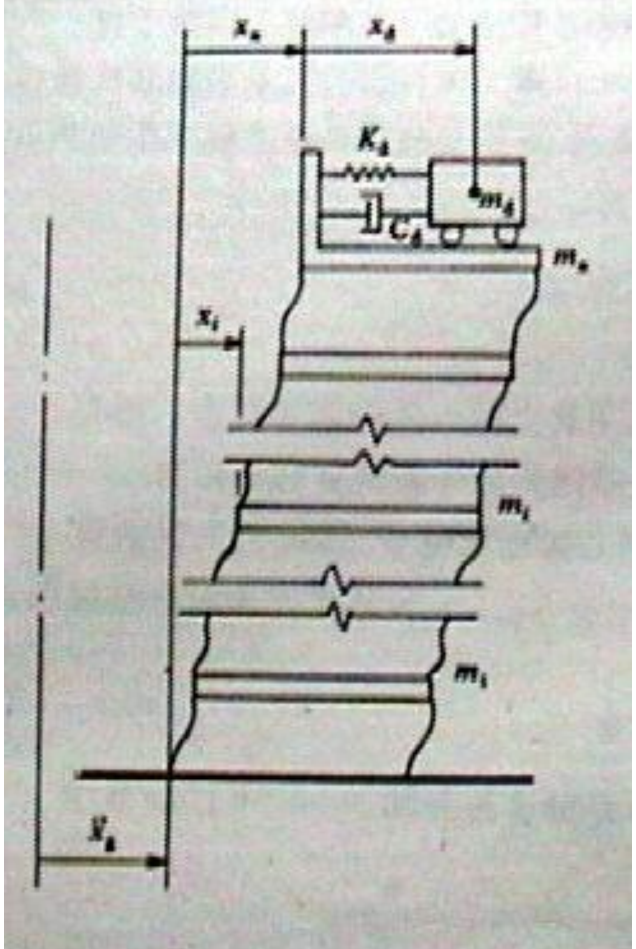


图 15-1-1 多质点结构被动调谐减震计算模型

Equation of motion for the MDOF system with TMD

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}-\left\{E\right\}(C_d\dot{x}_d+K_d x_d)=-[M]\{I\}\ddot{x}_g(t)$$

$$m_d(\ddot{x}_d+\ddot{x}_n)+C_d\dot{x}_d+K_d x_d=-m_d\ddot{x}_g$$

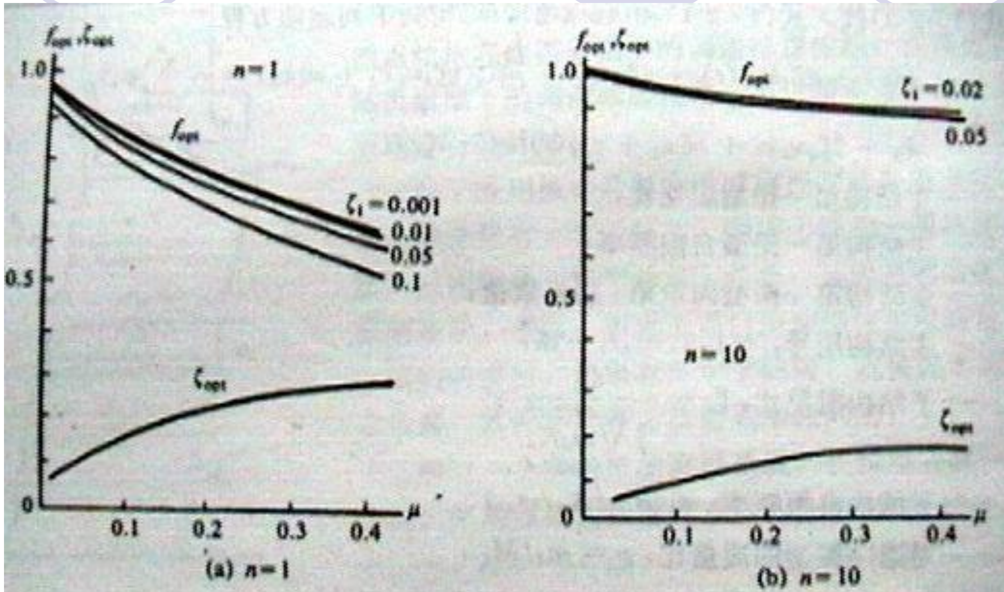
对多质点结构及上述运动方程，可运用振型分解法求解。

The above equations for MDOF system can be solved by using mode decomposition method

结构被动调谐减震控制体系，子结构设于顶层，以控制第一振型为主要目标。

The TMD is installed on the top of the MDOF system, mainly aiming at controlling the first mode of vibration

为了使主结构第一振型的位移反应得到明显的减震控制，设计时必须选取最优频率比和最优阻尼比。



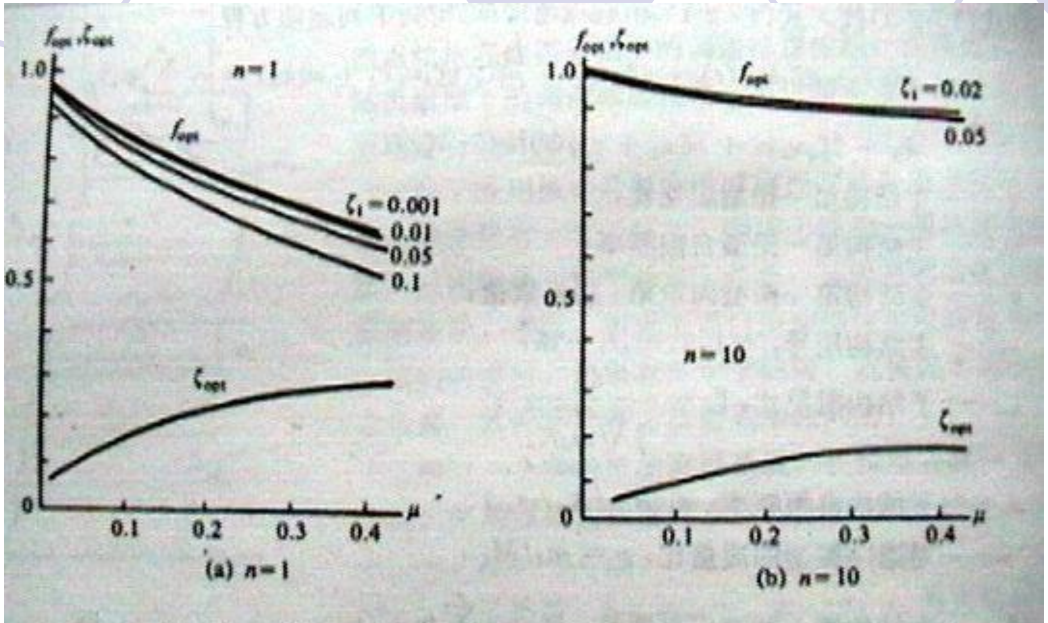
(a) $n=1$

(b) $n=10$

图 15-2-1　调谐减震结构不同自由度 n 时的最优频率比和阻尼比

从图可知，在子结构质量比μ相同的情况下，为了达到最优的减震效果，调谐结构体系的最优频率比$f_{opt}=\omega_d/\omega_1$和子结构最优阻尼比$\zeta_{opt}=C_d/(2\sqrt{m_d K_d})$随着结构自由度的增多，前者随之提高，后者降低。

In order to control the displacement of the first mode of the structure efficiently, the optimal frequency ratio and damping ratio must be selected in design.



(a) $n=1$ (b) $n=10$

图 15-2-1 调谐减震结构不同自由度 n 时的最优频率比和阻尼比

The figure shows that, if the mass ratio keeps a constant, in order to achieve the optimal vibration control, the optimal frequency ratio of the system, $f_{opt}=\omega_d/\omega_1$ increases with the degree of freedom of the structure while the optimal damping ratio of the substructure, $\zeta_{opt}=C_d/(2\sqrt{m_d K_d})$ reduces when the degree of freedom increases.

4.4 结构被动调谐减震控制的工程应用

Engineering application of structural passive tuned vibration control

结构被动调谐减震控制在国内外的工程应用简况如下：

The application status of passive tuned vibration control domestic and abroad is as follows

（1）Types of tuned substructures: two types of tuned substructures are widely used in engineering. One is tuned mass damper(the supported or suspended), the other is tuned liquid damper

（2）调谐减震（振）目标：早期工程应用的大多数被动调谐结构，是以抗风减震为主要目标。

The objectives of tuned vibration control: Aiming at passive control on structures under earthquake or wind by tuned vibration absorption

（3）调谐减振结构的结构形式：早期工程应用的调谐结构大多数是超高层建筑、高耸塔架、大跨度桥梁等。

Types of structures by tuned vibration control: early application of tuned vibration control are mostly super high-rise buildings, high-rise towers and large span bridges.

4.4 结构被动调谐减震控制的工程应用

Engineering application of structural passive tuned vibration control

（4）Application examples in China(including the proposed projects)：Shanghai TV Tower(TMD), Nanjing TV Tower (TLD) , Zhuhai Jinshan Building (TLD), Jiujiang Yangtze River Bridge(TLD）

（5）Engineering applications abroad

USA: 纽约花旗大楼 Citigroup building in New York (TMD) ,波士顿约翰汉考塔 John Hancock Tower in Boston (TMD)

Japan:千叶港观光塔 Chiba Port Tower (TMD), 福冈塔 Fukuoka Tower (TMD), 长崎机场塔 Nagasaki Airport Tower (TLD), 横滨港导航 塔Yokohama Port navigation tower (TLD), 新横滨王子饭店Shin Yokohama Prince Hotel (TLD).